Mathematics educators frequently extol the virtues of inquiry-based instruction to classroom teachers (Brahier 2008). Indeed, visions of motivated students collaboratively investigating mathematics tasks of their own design provide an appealing instructional picture for teachers. Although research suggests that inquiry benefits learners by allowing them to make sense of the mathematics they encounter, particularly through discourse with peers (Carpenter et al. 1989; Jaworski 2007), to assume that all (or even most) teachers successfully create and implement inquiry-oriented lessons in their classrooms is wishful thinking.

Teachers may incorrectly identify cookbook lessons—those that lead students through a series of procedural steps in a recipe-like fashion—as mathematical inquiry because students are active as they work through such tasks. Unfortunately, cookbook tasks give students few opportunities to develop their own methods of investigation or to realize the potential of mathematics as a creative area of study. Recognizing the unsatisfactory nature of recipe-oriented teaching materials, we share an approach we use with teachers to transform cookbook lessons into materials that more fully embrace the fundamental tenets of mathematical inquiry.

**PRINCIPLES OF INQUIRY TEACHING**

Inquiry-based teaching is firmly grounded in the scientific method. As such, the literature of science education provides a wealth of ideas for engaging mathematics students in inquiry-oriented experiences. For instance, in “Rethinking Laboratories,” Volkmann and Abell (2003) discuss strategies for transforming recipe-based science labs into inquiry-oriented labs. The authors encourage teachers to modify lessons according to four adaptation principles: questions, evidence, explanation, and communication. Following this recommendation, we have developed (and continue to refine) tools to assess the extent to which various teaching materials are inquiry oriented. For instance, the rubric in figure 1 allows teachers to reflect on the meaning and nature of inquiry in their classrooms as they plan daily instruction.

The four lesson criteria listed in the leftmost column of the rubric—task, analysis, revision, and presentation (TARP)—are based on Volkmann and Abell’s four adaptation principles, with explanation and communication collapsed into a single criterion: presentation. Because the notion of revision is arguably less familiar in secondary school mathematics classrooms than in science classes, we have amplified its importance with its own descriptor. The remaining columns of the rubric...
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The Triangle Centers task
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Assessing teaching materials with an inquiry rubric such as the one in figure 3 is helpful for teachers in at least two ways. First, rubrics encourage us to slow down and consider materials more carefully. For instance, what learning opportunities are afforded by the activity sheet? Do the materials actually support creative thinking and student autonomy? Second, the rubric we use encourages teachers to consider alternatives. For instance, what content connections could be added? In what context could the activity sheet tasks be presented?

FROM COOKBOOK TO INQUIRY
Using the Rubric to Brainstorm about Revisions
Once teachers determine that instructional materials fail to support inquiry, they use criteria from the TARP rubric to inform their revisions. Next we explore possible modifications of the Triangle Centers task along the four dimensions—task, analysis, revision, and presentation—articulated in the rubric.

Revising the Task
As teachers assess the Triangle Centers task, it is apparent that the activity sheet is little more than a checklist of procedures to be completed with technology. As such, it allows little room for multiple interpretations or solution strategies. Although several potentially interesting questions appear at the end of the activity sheet (e.g., does the circumcenter always lie within the triangle?), these are not student generated. Further, the tasks lack context, which spurs student interest as well as meaningful follow-up questions. Eliminating the checklist of procedures while framing the exploration as a problem with a meaningful context will lead to materials that better support inquiry.

Modifying the Analysis
Opportunities for student analysis are squelched by the checklist of procedures in the activity sheet. For instance, rather than uncovering the usefulness of perpendicular bisectors in the task, students are told to construct them. Moreover, the explicit mention of the term circumcenter at the end of the activity inhibits student exploration. As Wanko (2008) suggests, framing questions with embedded answers may lead to trivialization of the task.

Figure 4 illustrates a result obtained by searching for the phrase “circumcenter inside triangle” with a popular Internet search utility. Because it is so easy to find this kind of information on the Web, specific mathematics terminology directly related to the Triangle Centers task should be removed. Further, the checklist of student procedures should be eliminated.
Modifying the Revision

Opportunities for student revision are not explicitly incorporated into the Triangle Centers task activity sheet. For instance, students are not encouraged to explore variations of tasks—what, for instance, would happen if bisectors of angles rather than sides were constructed?—and they are not explicitly encouraged to “drag” vertices of their initial triangle when crafting answers to the questions at the end of the activity sheet. Revision lies at the heart of mathematical inquiry in interactive environments (De Villiers 1999). Modifying the activity sheet explicitly to encourage student revision is a must for teachers wishing to provide students with inquiry-oriented experiences in the investigation.

Modifying the Presentation

The final products generated by students in the original Triangle Centers task are twofold: a sketch constructed with interactive geometry software and written responses to two short questions. Although the activity sheet does not specifically mention the audience, the sole beneficiary of student work is likely the classroom teacher. Because the ways in which students can solve the activity sheet tasks are limited (the checklist of steps affords little variation or creativity), students have little incentive to share their problem-solving strategies with other students. Further, given the absence of a real-world context, it is unlikely that anyone other than the classroom teacher would be interested in such solutions.

As teachers encourage students’ presentation of work in a more inquiry-oriented manner, the reasons for sharing work should be compelling. Providing rich, contextual tasks with multiple solutions offers students an environment in which to share work.

REVISIGN MATERIALS WITH THE TARP RUBRIC

The Choosing a House Problem
Using our rubric to inform activity sheet modifications, we constructed the revision of the Triangle Centers task (see fig. 5).

Revising the Task
Before students solve the problem with technology, we provide a paper copy of the written problem and a map (see fig. 5) and ask them to make a conjecture about the location of a house equidistant from the three schools. This preliminary conjecture helps students keep a record of their initial construction ideas as they revise their solutions. Many students construct a point on the interior of the triangle formed by the three schools, as shown in figure 6.
When students are presented with this task, they typically recognize that a triangle center is involved; however, they do not remember or are not familiar with the circumcenter construction. We have found that students examine the construction tools available in the software menus to prompt their initial ideas of how to solve this problem. Students working in GeoGebra sometimes begin by constructing the medians of the triangle (see fig. 7).

Not until students measure the distance from the point of intersection of the three medians (the triangle’s centroid) to the three vertices are they convinced that their location for the family’s new house is not correct. Many students will revise their constructions and try to solve the problem using the capabilities of the software until they find the circumcenter of the triangle (see fig. 8). Students then communicate their geometric solution as well as their construction techniques in a written format as an e-mail message. Students also present their solution to classmates using construction protocol tools in GeoGebra (see fig. 9).

**Assessing the Revision**
The revision is more open-ended and provides a context for exploration. In the revision, students are not led through a series of technology steps; instead, they use their problem-solving skills to find a solution to the task. As the scored rubric in figure 10 indicates, the revised task is not completely student centered; however, it provides opportunities for students to test construction techniques, make conjectures, and communicate their understanding in multiple ways.

**AN EXAMPLE FROM ALGEBRA**
Cookbook examples are not restricted to interactive geometry software explorations. For virtually any mathematical subject and any technology, numerous examples of recipe-based teaching exist. The example in figure 11 highlights a second-year algebra lesson that encourages recipe-based work as students solve systems of equations. The activity sheet leads the student through the steps to produce...
With most any graphing calculator, you can solve a system of linear equations using MATRIX functions. An augmented matrix contains the coefficients of each equation with an extra column containing constant terms.

Consider the following example:

\[2x + y = 60\]
\[x + 2y = 72\]

**STEP 1:** Begin by creating a \(2 \times 3\) matrix.

**STEP 2:** Enter the augmented matrix into the calculator.

**STEP 3:** Calculate the reduced-row echelon form (rref) of the matrix. The first row represents \(x = 16\), and the second row represents \(y = 28\). The solution is \((16, 28)\).

Use an augmented matrix for each system. Solve with a graphing calculator.

1. \(2x + 1y = 90\)
   \(1x + 2y = 72\)

2. \(15x + 11y = 36\)
   \(4x + 3y = 18\)

3. \(2x + 4y = 5\)
   \(1x + 2y + 3z = 4\)
   \(3x + 3z = 4.5\)

A reduced-row echelon form (rref) of a coefficient matrix using a TI-Nspire graphing calculator.

Using the TARP rubric, we made the following notes about the activity sheet:

- The tasks lack context. When students complete problems 1–3 in the activity sheet, the exercises are solved the same way by all students—namely, by using the rref calculator function. The tasks are not sufficiently rich to support multiple solution strategies.
- Techniques for analyzing the exercises are wholly initiated by the materials. Students are instructed to use augmented matrices. This method is illustrated immediately before students are asked to solve problems 1–3.
- The materials offer few opportunities for critical thinking as students solve the exercises. Thus, students have few if any opportunities to form and revise conjectures through experimentation.

With these observations in mind, we can transform the materials into a more inquiry-based activity. **Figure 12** illustrates a task inspired and adapted from materials presented in *Mathematics in Context* (Kindt et al. 2006) for use with secondary school students.

Note that the revised task corresponds to problem 3 of the cookbook activity sheet. Unlike the original, the revision presents students with a real-world context for studying systems of equations—namely, determining the total cost of a food order at a sandwich shop. Further, it provides students with opportunities to explore a wider variety of solution strategies. For instance, students may begin by combining orders to calculate the unit cost for top-
pings; such an approach is illustrated in figure 13. Students taking such an approach are actually performing matrix row operations, although they may not recognize it as such. Continuing in this fashion, students may use paper-and-pencil methods to find the unit cost of meats and veggies, or they may use a mix of technology-oriented approaches (see figs. 14 and 15).

With the cost of each item known, calculating the cost of the entire order is a straightforward exercise. A spreadsheet-based approach is shown in figure 15.

The revised task provides students with a realistic context for exploring systems of equations while providing them with freedom to explore multiple solution methods in a manner more consistent with inquiry-based teaching methods. The TARP rubric was useful for framing this revision.
CONCLUSION
In this article, we provide a framework for transforming cookbook lessons into inquiry-based ones. Too often, the teaching materials that we use in our classrooms fail to allow our students to develop their own methods of investigation. In the examples given here, we used our framework—the TARP rubric—to consider materials with fresh eyes, providing students with revised tasks that better convey mathematics as a meaningful, creative area of study.

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REFERENCES


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Fig. 14 Once the cost of the topping is known, the unit cost of meats and veggies can be found.

Fig. 15 Students can define cost as a formula in a TI-Nspire spreadsheet (a), fill down the cost formula (b), and then use the sum function to determine the total cost of all orders (c).